Mathematics - 3 (Hons.)

Answer any six questions.

1. (a) State and prove Leibnitz's theorem to find the nth derivative of a product of two functions of x.

   (b) If $y=e^{a\sin^{-1}x}$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$.

2. (a) State and prove Euler's theorem on homogeneous functions of two independent variables.

   (b) Expand $e^{\sin x}$ as far as the term involving x^4.

3. (a) Find the condition that the line $x\cos\alpha+y\sin\alpha=p$ should touch the curve $x^m y^n=a^{m+n}$

   (b) Show that in the exponential curve $y=be^{x/a}$ the subtangent is of constant length and subnormal varies as the square of the ordinate.

4. (a) Find the radus of curvature for the pedal curve p = f (r).

   (b) For the curve $r^m=a^m\cos m\theta$, prove that $e=\dfrac{a^m}{(m+1)r^{m-1}}$

5. Evaluate any two of the following integrals :

   (a) $\int\dfrac{dx}{(x-3)\sqrt{x+1}}$ (b) $\int\dfrac{dx}{5+3\cos x}$

   (c) $\int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$ (d) $\int x^p\sin q^x\,dx$

6. Evaluate any two of the following :

   (a) $\displaystyle\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ (b) $\displaystyle\int_0^{\pi}\frac{dx}{a+b\cos x}(a>b>0)$

   (c) $\displaystyle\int_0^{\pi/2}\log\sin x\,dx$ (d) $\displaystyle\int_0^{\pi/2}\cos^n x\,dx$

7. (a) Find the area of the loop of the curve $ay^2=x^2(a-x)$

   (b) Find the area of the cardioid $r=a(1+\cos\theta)$.

8. (a) Find the perimeter of the loop of the curve $3ay^2 = x(x-a)^2$.

   (b) Find the volume of the solid generated by revolving the cardioid $r = a(1+\cos\theta)$ about the initial line.

9. (a) Obtain Lagrange's condition for Maxima or Minima of functions of two independent variables.

   (b) Prove that $B(m,n) = \dfrac{\overline{|(m)}\,\overline{|(n)}}{\overline{|(m+n)}}$

10. (a) Show that every convergent sequence is bounded.

    (b) Show that : $\lim\limits_{x\to\infty} n^{1/n} = 1$