UG(2) — M (Sub / Gen)

Sc. & Arts – New

2017

Time : 3 hours

Full Marks : 100

Pass Marks : 33

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

*Answer **eight** questions, selecting at least **one** from each Group.*

Group – A

1. (a) State and prove leibnitz's theorem to find the nth derivative of a product of two functions.

   (b) If $u = \cos^{-1}\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + \dfrac{1}{2}\cot u = 0$.

http://www.lnmuonline.com

2. (a) State and prove Maclaurin's series.

   (b) Evaluate :

$$\lim_{x\to\frac{\pi}{2}} (\sin x)^{\tan x}$$

3. (a) Find the length of subtangent and subnormal to the curve $y = f(x)$ and show that

$$\frac{\text{Sub tangent}}{\text{Sub normal}} = \left(\frac{\text{length of tangent}}{\text{length of normal}}\right)^2$$

   (b) Find the radius of curvature of a curve in pedal form.

4. (a) Find the limit when $n \to \infty$ of the series

$$\frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + \cdots\cdots + \frac{1}{8n}.$$

   (b) If $I_{m,n} = \int_0^{\pi/2} \cos^m x \cos nx \, dx$, show that

$$I_{m,n} = \frac{m}{m-n} I_{m-n,\,n+1}.$$

5. Find the area of the loop of the curve $x(x^2 + y^2) = a(x^2 - y^2)$.

6. (a) If $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x}\, dx$, then show that

$\Gamma(n) = (n-1)(n-2)\cdots 3.2.1\ \Gamma(1)$ and $\Gamma(1) = 1$.

(b) Evaluate $\iiint_R u^2 v^2 w\, du\, dv\, dw$ where R is the region $u^2 + v^2 \leq 1,\ 0 \leq w \leq 1$.

7. Solve any **two** of the following differential equations :

(a) $(x+y)^2 \frac{dy}{dx} = a^2$

(b) $(x^2 - y^2) \frac{dy}{dx} = 2xy$

(c) $(1 - x^2) \frac{dy}{dx} - xy = 1.$

http://www.lnmuonline.com

http://www.lnmuonline.com

http://www.lnmuonline.com

http://www.lnmuonline.com

(d) $x^2 \frac{dy}{dx} + xy = y^2$

8. (a) Solve any **one** of the following differential equation :

(i) $y = 2px + p^2$

(ii) $y = px + p - p^2$ where $p = \frac{dy}{dx}$

(b) Find the orthogonal trajectories of the curve $r = a\theta$.

9. Solve any **two** of the following differential equations :

(a) $\frac{d^2y}{dx^2} - y = \cos 2x$

(b) $\frac{d^2y}{dx^2} + 4y = \sin 3x + e^x + x^2$

(c) $\frac{d^2y}{dx^2} + y = xe^{2x}$

http://www.lnmuonline.com

http://www.lnmuonline.com

http://www.lnmuonline.com

Group – B

10. (a) Define scalar product of three vectors and show that the scalar triple product $\vec{a} \cdot (\vec{b} \times \vec{c})$ equals the volume of the parallelopiped formed by the three vectors originating from a common point.

(b) Prove that :

$$[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = 2\,[\vec{a}, \vec{b}, \vec{c}]$$

11. (a) Show that the necessary and sufficient condition for the vector function $\vec{a}$ (t) to have constant magnitude is $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$.

(b) If $\vec{a} = t^2\vec{i} + (3t^2 - 2t)\vec{y} + \left(2t - \frac{1}{t}\right)\vec{k}$, then find $\frac{d\vec{a}}{dt}$ when t = 1.

12. (a) If ϕ and $\vec{a}$ are continuously differentiable scalor and vector point functions respectively, then prove that curl $(\phi\,\vec{a}) = \phi\,(\text{curl }\vec{a}) + (\text{grad }\phi) \times \vec{a}$.

http://www.lnmuonline.com

http://www.lnmuonline.com

(b) Prove that :

curl (grad ϕ) = 0

Group – C

13. (a) Obtain the general condition of equilibrium of a system of forces acting in one plane upon a rigid body.

(b) Show that any system of forces, acting in one plane upon a rigid body can be reduced to either a single force or a single couple.

14. (a) State and prove the principle of virtual work for any system of forces in one plane.

(b) Six equal rods AB, BC, CD, DE, EF and FA are each of weight W and are freely joined at their extremities so as to form a hexagon; the rod AB is fixed in horizontal position and the middle points of AB and DE are joined by a string; prove that its tension is 3W.

http://www.lnmuonline.com

15. A particle moves is a straight line OA starting from rest at A and moving with an acceleration which is always directed towards O and varies as the distance from O, discuss the motion.

16. Obtain the tangential and normal components of velocity and acceleration of a particle moving along a curve in a plane.