HG (3) – Math (8) Num. Th. (Sc & Arts)

2021

Time : 3 Hours

Maximum Marks : 70

Candidates are required to give their answers in their own words as far as practicable.

Answer any five questions

D-262

1. (a) State and prove Fundamental theorem of Arithmetic.

   (b) If a/bc and (a,b)=1 then prove that a/c.

2. (a) State and prove Chinese remainder theorem for the solution of simultaneous congruence.

   (b) Solve :

   $103x = 57(mod\ 211)$

3. (a) Discuss the solvability of the linear congruence

   $ax \equiv b(mod\ m)$.

   (b) Find the remainder when 2^{340} is divided by 341.

4. (a) State and prove Euler's theorem of congruence.

   (b) Find the highest power of p that is a factor of $\lfloor n$

5. (a) Prove that the quadratic congruence

   $x^2 \equiv -1(mod\ p), p$ is prime has a solution if and only if $p \equiv 1(mod\ 4)$.

   (b) Show that $x^7 - n$ is divisible by 42.

6. (a) If p is odd prime then prove that $\sum_{n=1}^{p-1}\left(\dfrac{n}{p}\right) = 0$

   (b) Solve the congruence $x^2 \equiv 5(mod\ 29)$.

7. (a) If p is prime, show that there exist no positive integers a and b such that $a^2 = pb^2$

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(b) Prove that a prime number $p = 4n+1$ can be expressed as sum of two squares.

8. (a) Establish mobius inversion formula.

(b) Find the quadratic residues modulo 17.

9. (a) Obtain the positive integral solution of $x^2 + 2y^2 = z^2$

(b) Express 3185 as a sum of two squares.

10. Write notes on any two of the following :

(a) Euler partition theorem

(b) Quadratic reciprocity law

(c) Fermat's last theorem

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